glass tube, have a comparatively feeble catalysing power; fractured surfaces, however, cause the recombination of the atoms, and are strongly heated.

The action of water-vapour or oxygen in enhancing the Balmer spectrum, and suppressing the secondary spectrum of hydrogen, is probably due to its action on the wall of the tube, which, when dry, catalyses the atomic hydrogen as fast as it is formed by the current.

The peculiar spectroscopic phenomena observed with long hydrogen tubes, and described in an earlier paper, are explained.

Methods are suggested for determining the physical and optical properties of atomic hydrogen gas.

On the Scattering of β-Rays.

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The scattering of β-rays by thin sheets of matter has been the subject of a number of experimental and theoretical investigations in recent years. J. A. Crowther and B. F. Schonland* give a good summary of the principal results so far obtained, together with an account of a new series of measurements.

Crowther and Schonland regard their experiments as decisively in favour of the theory of "single scattering," which was shown by Rutherford to account for the scattering of α-rays by thin plates in a thoroughly satisfactory way. In the case of α-rays the fraction of the rays scattered through considerable angles is very small, and it is on this that the success of the theory depends. In the case of β-rays the fraction scattered is very much larger, and it is difficult to believe that "multiple scattering" can be neglected.

On Rutherford's theory of single scattering each ray is supposed to be deflected by a single collision with an atom. The number of collisions is clearly proportional to the thickness (t) of the sheet, so that the fraction scattered through more than a given angle is also proportional to the thickness and

$$I/I_0 = 1-kt, \tag{1}$$

where I denotes the number of rays deflected through less than a given angle, I_0 the number of rays incident on the plate, and k is a constant.

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* 'Roy. Soc. Proc.,' A, vol. 100, p. 526 (February, 1922).

The constant k is found to be proportional to the square of the cotangent of the scattering angle (ϕ), so that if we take this angle very small, equation (1) makes I/I_0 large and negative, which is impossible. This equation therefore only applies to large angles of scattering. Small deflections are neglected on the single scattering theory.

So long as I and I_0 remain nearly equal and ϕ is not too small, equation (1) is satisfactory, but it is easy to see that it must be replaced by

$$I/I_0 = \epsilon^{-kt} \tag{2}$$

when I differs appreciably from I_0. Thus in one of Crowther and Schonland's experiments two gold foils scattered about 30 per cent. of the β-rays, and one foil scattered about 14 per cent. Now, if the first foil scatters 14 per cent. of the incident rays, then only 86 per cent. remain to be scattered by the second foil, so that we should expect the second foil to scatter 14 per cent. multiplied by 86/100, or 12 per cent. and not 14 per cent. Of course, the 14 per cent. scattered by the first foil passes through the second foil, and we should expect 14 per cent. of these, or 2 per cent., to be scattered a second time. This second scattering, however, is, roughly speaking, as likely to decrease the deflections as to increase them, and so can be neglected.

It is clear that when a considerable percentage of the rays is scattered then the exponential equation will represent the single scattering theory better than the linear equation. The two equations, of course, become identical for large values of ϕ when the fraction of the rays scattered is very small, as is the case with α-rays. Equation (2) also has the advantage that it makes I/I_0 equal to zero when $\phi = 0$, which is clearly correct.

When a β-ray passes through a thin sheet we should expect it to make a large number of collisions with atoms, so that we should expect scattering, due to the cumulative effect of many deflections. This idea is the basis of the "multiple scattering" theory which was put forward by J. J. Thomson in 1910. J. J. Thomson based his calculations on the theory that an atom consists of a solid sphere of positive electricity, inside which negative electrons move about, but the equation obtained is independent in form of any special theory as to atomic constitution. J. J. Thomson obtained the equation

$$I/I_0 = 1-\epsilon^{-\phi^2/kt}, \tag{3}$$

where I/I_0 is the fraction of the β-rays remaining inside a scattering angle ϕ after passing through a plate of thickness t.

Crowther and Schonland considered that their results were decisively in favour of equation (1) as against equation (3), and conclude that the "single scattering" theory represents the scattering of β-rays when I/I_0 is greater than one-half.

According to what has been said, the equations which represent the two theories should be (2) and (3) and not (1) and (3), so I have compared the results given by Crowther and Schonland for gold with (2) and (3).

To do this the values of $t^{-1} \log I_0/I$ and $t \log I_0/(I_0 - I)$ were calculated.

(1) I/I_0. (Obs.)	(2) t.	(3) $t^{-1} \log I_0/I$.	(4) I/I_0. (Calc.)	(5) $t \log I_0/(I_0-I)$.	(6) I/I_0. (Calc.)
0·86	1	0·064	0·83	0·85	0·89
0·67	2	0·087	0·69	0·96	0·67
0·52	3	0·094	0·57	0·96	0·52
0·46	4	0·084	0·47	1·07	0·43
0·38	5	0·084	0·39	1·03	0·36

The first column of the above table contains the values of I/I_0 obtained from the curve given by Crowther and Schonland for $\phi = 0{\cdot}11$. The second column gives the number of gold foils through which the β-rays were passed. The numbers in the third column should be constant if equation (2) represents the results, and the numbers in the fifth column should be constant if equation (3) represents the results. It appears that either of the two equations can be used to represent the results about equally well.

The mean value of $t \log I_0/(I_0 - I)$ is 0·97, and if we put this value in the equation (3) of the multiple scattering theory and use it to calculate the values of I/I_0, we get the numbers given in the last column of the table. The mean value of $t^{-1} \log I_0/I$ is 0·0826, and using this with equation (2) to calculate I/I_0 we get the numbers given in the fourth column.

Both sets of calculated values agree fairly well with the observed values given in the first column, but the numbers calculated by the equation of the multiple scattering theory agree rather better with the observations than those calculated by the single scattering equation.

It seems quite clear that these results cannot be regarded as disproving the multiple scattering theory.

The following table contains a similar comparison of Crowther and Schonland's results with $\phi = 0{\cdot}145$ with equations (2) and (3):—

I/I_0. (Obs.)	t.	$t^{-1} \log I_0/I$.	$t \log I_0/(I_0-I)$.	I/I_0. (Calc.)
0·90	1	0·046	1·00	0·97
0·82	2	0·043	1·44	0·81
0·72	3	0·048	1·66	0·67
0·61	4	0·054	1·63	0·57
0·51	5	0·061	1·55	0·49

In this case the agreement between the numbers in the last column, calculated by equation (3), with the observed values, is not as good as in the first case; but equation (2) also does not represent the observations very well, as is clear from the way in which the values of $t^{-1} \log I_0/I$ increase with t.

The values of I/I_0 in this case agree nearly with equation (1), which does not represent either of the two theories, except when I/I_0 is very nearly equal to unity. It seems fair to conclude that these results do not enable us to decide between the two theories, but that they agree rather better with the multiple scattering theory than with the single scattering theory.

In view of this, it seemed to the writer worth while to work out the theory of multiple scattering of β-rays on the basis of Rutherford's nucleus theory of the atom, and to compare Crowther and Schonland's results with this theory as well as with the single scattering theory.

The theory of the motion of an electron near a positive nucleus has been worked out by C. G. Darwin, on the assumption that the total energy of the electron, when moving with a velocity v, is equal to $m(1-\beta)^{-\frac{1}{2}}$, where m is its energy when $v = 0$ and is assumed constant, $\beta = v/c$, and c denotes the velocity of light. This formula gives the variation of the energy with v when the electron is far from other charged bodies, but when it is very near the positively charged nucleus, m is diminished and so does not remain constant.

Suppose an electron, consisting of a sphere of electricity of amount e and radius a, is at rest at a distance r from a positive nucleus, consisting of a sphere of electricity of amount E and radius b. The electrostatic energy of the system is then

$$\frac{1}{2}e\left(\frac{e}{a}+\frac{E}{r}\right)+\frac{1}{2}E\left(\frac{E}{b}+\frac{e}{r}\right).$$

In addition there is the internal energy of the charges which is $\frac{1}{6}\frac{e^2}{a}+\frac{1}{6}\frac{E^2}{b}$. The total energy is therefore

$$\frac{2}{3}\frac{e^2}{a}+\frac{2}{3}\frac{E^2}{b}+\frac{eE}{r}.$$

The energy $\frac{2}{3}\frac{e^2}{a}$ is located very close to the electron since its density varies inversely as the fourth power of the distance and the same is true of $\frac{2}{3}\frac{E^2}{b}$. The energy eE/r, however, is distributed in a symmetrical manner about the two charges, and its density very near either of them varies inversely as the square of the distance. This energy cannot therefore be regarded as moving with the electron like the energy $\frac{2}{3}\frac{e^2}{a}$.

When r is small most of the energy Ee/r is near the charges so that since its distribution is symmetrical it may be approximately true to suppose one-half of it to be associated with the electron and to move with it, and the other half to be associated with the nucleus.

Making this approximate assumption we have the energy of the electron, when it is at rest, equal to $\frac{2}{3}\frac{e^2}{a}+\frac{\mathrm{E}e}{2r}$, so that when the electron is moving with a velocity v its energy may be taken to be

$$\left(\frac{2}{3}\frac{e^2}{a}+\frac{\mathrm{E}e}{2r}\right)(1-\beta^2)^{-\frac{1}{2}}.$$

Suppose an electron starts towards a nucleus from a great distance with an initial velocity v_0 so that its initial energy is $m(1-\beta_0{}^2)^{-\frac{1}{2}}$ where $m=\frac{2}{3}\frac{e^2}{a}$ and $\beta_0=v_0/c$.

Then assuming the total energy remains constant and that the nucleus is fixed we have

$$m(1-\beta_0{}^2)^{-\frac{1}{2}}=\left(m+\frac{\mathrm{E}e}{2r}\right)(1-\beta^2)^{-\frac{1}{2}}+\frac{\mathrm{E}e}{2r}.$$

The kinetic energy of the electron is equal to

$$\left(m+\frac{\mathrm{E}e}{2r}\right)((1-\beta^2)^{-\frac{1}{2}}-1)=m((1-\beta_0{}^2)^{-\frac{1}{2}}-1)-\frac{\mathrm{E}e}{r}.$$

Thus the kinetic energy on this theory has the same value as on the usual form of the theory.

The angular momentum of the electron about the nucleus is equal to

$$\frac{m+l}{c^2\sqrt{(1-\beta^2)}}\,r^2\dot{\theta}=p,$$

where $l=\mathrm{E}e/2r$, and θ is the angle r makes with a fixed radius in the plane of the orbit.

The energy equation gives

$$\sqrt{(1-\beta^2)}=\frac{m+l}{m'-l}$$

where $m'=m(1-\beta_0{}^2)^{-\frac{1}{2}}$ so that

$$p=\frac{1}{c^2}(m'-l)\,r^2\dot{\theta}.$$

On working out the equation of the orbit in the usual way we obtain

$$u=-\frac{\mathrm{E}e}{2p^2c^2}(m+m')(1+\epsilon\cos\theta)$$

where $u=r^{-1}$ and ϵ denotes the eccentricity. Thus, the orbit in the case considered will be a hyperbola, just as though the mass of the electron remained constant.

Thus, on the present theory, the complications of the usual theory do not appear even when the velocity approaches that of light.

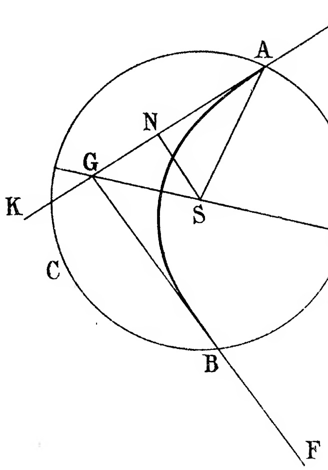
The kinetic energy of the electron has the usual value, but its total energy is equal to $m(1-\beta_0^2)^{-\frac{1}{2}}-\mathrm{E}e/2r$ or $(m+\mathrm{E}e/2r)(1-\beta^2)^{-\frac{1}{2}}$ instead of $m(1-\beta^2)^{-\frac{1}{2}}$.

According to J. J. Thomson's theory of multiple scattering the number of electrons remaining inside an angle ϕ is given by $\mathrm{I}/\mathrm{I}_0 = 1-\epsilon^{-\phi^2/kt}$ where kt is equal to the average value of the sum of the squares of all the deflections of a β-ray while passing through a plate of thickness t.

If we assume that each atom consists of a nucleus with a positive charge, equal to the charge on one electron multiplied by the atomic number N, surrounded by N electrons, and neglect the field due to the electrons, then the scattering on the multiple theory comes out very much greater than that observed. It is clear that the electric field of each nucleus cannot be supposed to extend to infinity, but must be regarded as extending only as far as the surrounding electrons. The electrons limit the field of the nucleus and so diminish the scattering.

To get an approximate theory of scattering I have assumed each atom to consist of a positive nucleus with a charge Ne surrounded by a hollow sphere of negative electricity of amount Ne and radius R. The field inside R is then that due to the nucleus, and outside R it is zero.

Actually the electrons are probably arranged in several layers of different radii. The value of R obtained by a comparison of the theory with the observed scattering will, therefore, be a sort of average value. Also if some of the electrons are very near to the nucleus the β-rays may not pass inside them, so that the scattering may be the same as that due to an atom with a nuclear charge less than Ne.



In the figure let the nucleus be at S, and let the circle ABC represent the sphere of negative electricity of radius R. Also let EABF be the path of a β-ray. Let EA and FB produced meet at G, so that the angle FGK is the angle of deflection ϕ. Let SN be perpendicular to EK, and let $\mathrm{SN} = h$.

Let the equation of the path inside the sphere be $\mathrm{L}r^{-1} = 1+\epsilon\cos\theta$, so that the equation of EA, which is the tangent to the conic at A, is $\mathrm{L}r^{-1} = \epsilon\cos\theta+\cos(\theta-\alpha-\phi/2)$, where α denotes the angle ASN.

At A, $r = \mathrm{R}$ so that $\mathrm{LR}^{-1} = \epsilon\cos(\alpha+\phi/2)+1$. If $r = \infty$ we get $\theta = \frac{\phi}{2}+\frac{\pi}{2}$ and

$$0 = \epsilon\cos\left(\frac{\phi}{2}+\frac{\pi}{2}\right)+\cos\left(\frac{\phi}{2}+\frac{\pi}{2}-\alpha-\frac{\phi}{2}\right) \quad \text{or} \quad \epsilon = \frac{\sin\alpha}{\sin\phi/2}.$$

Hence
$$\mathrm{LR}^{-1} = 1+\frac{\sin\alpha}{\sin\phi/2}\cos(\alpha+\phi/2).$$

But $\cos\alpha = h/\mathrm{R}$ so that
$$\mathrm{L} = \frac{h^2}{\mathrm{R}}+h\sqrt{\left(1-\frac{h^2}{\mathrm{R}^2}\right)}\cot\frac{\phi}{2}. \tag{4}$$

Now $\mathrm{L} = -2p^2c^2/\mathrm{E}e(m+m')$, where m' denotes the total energy of the electron at $r = \infty$ in the hyperbolic orbit. The initial energy of the electron when the nucleus is surrounded by a negative sphere is equal to the energy in the orbit at $r = \mathrm{R}$, and so is given by
$$m'' = m'-l'$$
where $l' = \mathrm{E}e/2\mathrm{R}$.

Hence
$$\mathrm{L} = \frac{-2p^2c^2}{\mathrm{E}e(m+m''+l')}$$
and
$$p = m''v'h/c^2,$$
where v' denotes the initial velocity of the β-ray, so that its initial momentum is $m''v'/c^2$.

Hence
$$\mathrm{L} = \frac{-2m''^2v'^2h^2}{\mathrm{E}e(m+m''+l')c^2} = -\frac{2\mathrm{T}h^2}{\mathrm{E}e}$$
where
$$\mathrm{T} = \frac{m''^2-m^2}{m+m''+\mathrm{E}e/2\mathrm{R}}.$$

Unless R is less than 10^{-11} cm., $\mathrm{E}e/2\mathrm{R}$ is negligible, so that $\mathrm{T} = m''-m$, which is the initial kinetic energy of the β-ray.

Let
$$k = -\mathrm{E}e/2\mathrm{T} \text{ so that } \mathrm{L} = h^2/k.$$
With (4) this gives
$$h = \left\{\frac{1}{\mathrm{R}^2}+\left(\frac{1}{k}-\frac{1}{\mathrm{R}}\right)^2\tan^2\frac{\phi}{2}\right\}^{-\frac{1}{2}}. \tag{5}$$

When a β-ray passes through a plate of thickness t, containing n atoms per unit volume, the number of collisions it makes for which h is between h and $h+dh$ is $2\pi nthdh$. If ϕ is the deflection corresponding to h, then the sum of all the values of ϕ^2 is given by

$$\Sigma\phi^2 = \int_0^R 2\pi nth\phi^2 dh.$$

Substituting the value found for h and putting $\frac{1}{2}\phi = x$, we get

$$\Sigma\phi^2 = 8\pi nta^2R^2 \int_0^{\pi/2} \frac{x^2 \sin x \cos x \, dx}{(\cos^2 x + a^2 \sin^2 x)^2}$$

where $a = \frac{R}{k} - 1$.

Integrating by parts we get

$$\Sigma\phi^2 = \frac{8\pi nta^2R^2}{a^2-1} \left\{ \int_0^{\pi/2} \frac{xdx}{\cos^2 x + a^2 \sin^2 x} - \frac{\pi^2}{8a^2} \right\}.$$

Also $$\int_0^{\pi/2} \frac{xdx}{\cos^2 x + a^2 \sin^2 x} = \frac{\log a + 1}{a^2} + \frac{\log a + \frac{1}{3}}{3a^4} + \ldots.$$[*]

When a is large compared with unity we have therefore

$$\Sigma\phi^2 = \frac{8\pi ntR^2}{a^2} \left\{ \log a + 1 - \frac{\pi^2}{8} \right\}.$$

Putting in $a = \frac{R}{k}$ and $k = -\frac{Ee}{2T} = \frac{Ne^2}{2T}$ we get

$$\Sigma\phi^2 = 2\pi nt \left(\frac{Ne^2}{T}\right)^2 \left\{ \log R + \log \frac{2T}{Ne^2} + 1 - \frac{\pi^2}{8} \right\}.$$

Then according to the multiple theory of scattering

$$I/I_0 = 1 - \epsilon^{-\phi^2/\Sigma\phi^2}.$$

An examination of the integral in the expression for $\Sigma\phi^2$ shows that on this theory when a is large nearly all the scattering is due to the numerous small deflections. Large single deflections are so rare that they contribute little to the value of $\Sigma\phi^2$.

When t is very small the number of collisions will be small, so that $\Sigma\phi^2$ will not be equal to the calculated value which has been got by supposing that there are very many collisions corresponding to the values of h between h and $h+dh$. But the average value of $\Sigma\phi^2$ for a large number of β-rays will still be equal to the calculated value.

The value of $\Sigma\phi^2/t$ will now be calculated for β-rays passing through gold. The density of gold is 19·33, and its atomic weight 197·2, so that, since the mass of one atom of hydrogen is $1{\cdot}65\times 10^{-24}$ grm., we have for the number of atoms of gold per cubic centimetre

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[*] I am indebted to Prof. P. J. Daniell, of the Rice Institute, for the evaluation of this integral.

$$n = \frac{19{\cdot}33}{197{\cdot}2\times 1{\cdot}65\times 10^{-24}} = 5{\cdot}94\times 10^{22}.$$

The kinetic energy of the β-rays used by Crowther and Schonland was that due to a potential difference of $4{\cdot}59\times 10^5$ volts, or $1{\cdot}53\times 10^3$ electrostatic units.

For gold, $N = 79$, which, with $e = 4{\cdot}78\times 10^{-10}$, gives $(Ne^2/2T)^2 = 1{\cdot}53\times 10^{-22}$. Hence, if $\Sigma\phi^2/t = K$, then

$$K = 228{\cdot}4\,\{24{\cdot}97+2{\cdot}30\log_{10} R\}. \tag{6}$$

The average value of K given by Crowther and Schonland's results used earlier in this paper can be easily calculated.

With $\phi = 0{\cdot}11$, their results give $0{\cdot}97$ for the mean value of $t\log I_0/(I_0-I)$. This gives

$$K = \frac{(0{\cdot}11)^2}{8{\cdot}48\times 10^{-6}\times 0{\cdot}97\times 2{\cdot}303} = 636$$

where $8{\cdot}48\times 10^{-6}$ is the thickness of each gold leaf. With $\phi = 0{\cdot}145$ in the same way,

$$K = 736.$$

The mean of these two values is 686. Crowther and Schonland also give the value of ϕ^2/t_m for these rays for gold, where t_m is the thickness which scatters half the rays. With I/I_0 equal to one half, we have

$$K = \frac{\phi^2}{\log 2\,.\,t_m},$$

Crowther and Schonland give $\phi^2/t_m = 470$, which gives $K = 680$, and so agrees nearly with the value calculated directly from their values of I/I_0.

If we assume $K = 683$, we can calculate R by means of equation (6). The result is $R = 2{\cdot}2\times 10^{-10}$ cm. With this value of R, the theory of multiple scattering agrees with the scattering observed by Crowther and Schonland.

This value of R is probably too small, since the radius of an atom of gold is probably about 10^{-8} cm. It is probable, therefore, that an appreciable fraction of the electrons in the atom are inside most of the orbits of the β-rays. Let f denote the fraction of the electrons which are outside the orbits, so that fNe is the effective value of the nuclear charge. Then we have by (6)

$$K = 228{\cdot}4f^2\,\{24{\cdot}97+2{\cdot}30\log_{10} R-2{\cdot}30\log_{10} f\}.$$

With $K = 683$ this gives

$$\log_{10} R = -10{\cdot}84+\frac{1\,.\,3}{f^2}+\log_{10} f.$$

The following Table gives some values of f and the corresponding values of R:—

f.	R.
1	$2{\cdot}2 \times 10^{-10}$
0·7	$3{\cdot}3 \times 10^{-9}$
0·65	$1{\cdot}1 \times 10^{-8}$
0·60	$3{\cdot}6 \times 10^{-8}$
0·50	$1{\cdot}2 \times 10^{-6}$

Thus it appears that R varies very rapidly with f, and is equal to about 10^{-8} cm. when $f = 0{\cdot}65$.

When $f = 0{\cdot}65$ the effective nuclear charge is $51e$. If, then, we suppose that 51 of the electrons are outside the β-ray orbits at an average distance from the nucleus of 10^{-8} cm. and the remaining 28 inside the orbits, the theory accounts for the observed scattering. This seems to indicate that most of the electrons are near the surface of the atom, for the greater part of the scattering is due to collisions in which h is nearly equal to R, so that the deflection is small.

It seems clear that the theory of multiple scattering is adequate to account for the facts considered.

The single scattering theory will now be discussed. The number of collisions for which h is less than a given value made by a β-ray in passing through a plate of thickness t is equal to $\pi h^2 nt$, where n is the number of atoms per unit volume. In the case of single scattering through considerable angles the influence of the sphere of negative electricity becomes negligible, so that equation (6) reduces to

$$h = -\frac{\mathrm{E}e}{2\mathrm{T}} \cot \frac{\phi}{2}$$

which is the same as the equation used by Rutherford for α-rays.

If I_0 denotes the intensity of the incident rays, then

$$\mathrm{I}/\mathrm{I}_0 = \epsilon^{-\pi nt\,(\mathrm{E}e/2\mathrm{T})^2 \cot^2 \phi/2} = \epsilon^{-k't}.$$

The mean value of $t^{-1} \log \mathrm{I}_0/\mathrm{I}$ got from Crowther and Schonland's results for gold with $\phi = 0{\cdot}11$ is 0·0826, which gives $k'\phi^2 = 271$. Substituting the values of n, E, e, and T, we get

$$k'\phi^2 = \frac{\pi n \mathrm{N}^2 e^4}{\mathrm{T}^2} = 114.$$

If we suppose that the effective value of the nuclear charge is $f\mathrm{N}e$, then

$$f^2 = 271/114, \text{ so that } f = 1{\cdot}54.$$

Thus it appears that the observed scattering is considerably greater than that calculated on the single scattering theory.

The two theories, of course, are not mutually exclusive. The single scattering theory is clearly correct for very thin plates and large scattering angles, and the multiple scattering theory is clearly correct for thick plates and small scattering angles. It ought to be possible to work out a theory which would apply to plates of any thickness and to any scattering angles.

The multiple scattering theory in the case of rather thin plates does not take into account the large deflections, and the single scattering theory neglects the small deflections. It seems, therefore, that a more complete theory can be obtained by taking the product of the expressions for I/I_0 on the two theories, so that

$$I/I_0 = (1 - \epsilon^{-\phi^2/kt})\,\epsilon^{-k't\cot^2\phi/2}$$

where $$k = 2\pi n f^2 \left(\frac{Ne^2}{T}\right)^2 \left\{\log\frac{2TR}{fNe^2} + 1 - \frac{\pi^2}{8}\right\}$$

and $$k' = \tfrac{1}{4}\pi n f^2 (Ne^2/T)^2.$$

This equation would evidently agree with Crowther and Schonland's experiments, but would require a slightly smaller value of the factor f than 0·65. It is clear that the single scattering theory is only applicable when the factor $1 - \epsilon^{-\phi^2/kt}$ does not differ appreciably from unity. When the multiple scattering factor is one half the single scattering factor is about 0·9, so that the multiple scattering is then much more important than the single scattering.

Taking $f = 0{\cdot}6$ and $R = 1{\cdot}1 \times 10^{-8}$, the following values of the two factors in the above expression for I/I_0 are obtained.

t.	ϵ^{-3400t}.	$1 - \epsilon^{-0{\cdot}0000205/t}$	I/I_0.
1×10^{-6}	0·997	1·00	0·997
3	0·990	1·00	0·990
10	0·966	0·87	0·84
30	0·903	0·50	0·45
100	0·712	0·19	0·13

The first column gives the thickness of gold in centimetres. The second column gives values of ϵ^{-3400t} which is the single scattering factor for gold when $\phi = 0{\cdot}11$. The third column gives the multiple scattering factor for the same case, and the last column the product of the two factors. Thus, for thicknesses up to 3×10^{-6} cm. multiple scattering can be neglected, but with thicknesses above 10^{-5} cm., the multiple scattering is greater than the single scattering. The values of I/I_0 in the last column indicate an amount of scattering nearly equal to that observed by Crowther and Schonland for gold with $\phi = 0{\cdot}11$.

It appears that when both single and multiple scattering are taken into account, then if we assume $f = 0{\cdot}6$ and $R = 1{\cdot}1 \times 10^{-8}$ the theory agrees with the observations.

In an earlier paper* Crowther described a number of measurements of the scattering of β-rays, and obtained values of $\phi/\sqrt{(t_m)}$ nearly double those found for the heavier elements in the more recent work. In the earlier work larger scattering angles were used, and Crowther and Schonland state that they have recently found that with larger angles the scattering increases, and becomes equal to that previously observed by Crowther.

Since, on the theory proposed in this paper, the scattering for gold observed by Crowther and Schonland can be explained by taking $f = 0{\cdot}6$, it follows that the larger scattering observed by Crowther, and recently confirmed, will require f to be nearly unity.

We may suppose that, as the scattering angle is increased, the β-rays approach nearer to the nucleus, and so get inside more of the electrons, so that the effective nuclear charge increases.

With the single scattering theory employed by Crowther and Schonland, the scattering calculated for gold was nearly equal to that observed in the more recent experiments, and less than half that observed in the earlier experiments, although the effective nuclear charge was supposed to have its full value, so that they could not explain the increase of the scattering with ϕ as being due to an increase in the effective nuclear charge.

It appears that the theory proposed in the present paper is supported by Crowther and Schonland's experiments. If the usual form of the theory of the motion of an electron near a positive nucleus is used, instead of the modified theory suggested here, the calculated scattering is increased, so that the factor f has to be taken smaller in order to account for the facts.

The modified theory has the advantage of simplicity, and may be approximately true for electrons moving with high velocities when near the nucleus, but the success of the Bohr-Sommerfeld theory of spectra shows that the modified theory is not applicable to electrons moving comparatively slowly. For the purpose of explaining the scattering of β-rays the modified theory appears adequate, and I used it here because of its simplicity, since it is clear that the usual theory and the modified theory explain the facts equally well and merely require different values of the factor f.

Crowther and Schonland state that in the case of aluminium and copper their results agree slightly better with the assumption that the mass of the electron is constant than with the theory that it varies with its velocity as required by the theory of relativity. The results of the modified theory used here are the same, as regards scattering, as those of the theory of constant mass so that Crowther and Schonland's results appear to support the modified theory to some extent.

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* 'Roy. Soc. Proc.,' A, vol. 84, p. 226 (1910).